ON THE INFLUENCE OF THE NONSPHERICITY OF THE EARTH ON THE OPERATION OF A GYROHORIZON COMPASS

(O VLIIANII NESFERICHNOSTI ZEMLI NA RABOTU GIROGORIZONTKOMPASA)

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Merkin [1] mentioned conditions for the imperturbability of a gyrohorizon compass, taking account of the flattening of the earth. The force of attraction was assumed to be directed exactly towards the center of the earth. An unperturbed gyrohorizon compass is a gyro frame which continuously indicates (under appropriate initial conditions) the true local vertical and the plane of the meridian (to the accuracy of a course correction), i.e. the whole analysis is related to gravity and not to the central force. Hence, the assumption mentioned does not affect the basic result of the investigation.

It is shown below that the Geckeler gyrohorizon whose theory was given by Ishlinskii [2] is related in a definite manner to the direction of the central force for any hypothesis regarding the shape of the earth. Hence, even in an investigation of the influence of the flattening of the earth on the operation of a gyrohorizon it is more natural to use the method of taking account of the forces acting on a material system moving near the earth, as expounded in [2-5]. The deviation of the force of attraction from the direction towards the center of the earth is hence taken into account. Imperturbability conditions, understood to be the conservation by the gyrohorizon of the direction of the force of attraction and the plane of the meridian (to the accuracy of a course correction), as well as estimates of the errors induced in the operation of a Geckeler gyrohorizon by the deviation of the earth's shape from the spherical, are obtained.

1. Let a two-gyroscope compass rest on the surface of a body rotating with constant velocity U, whose force of attraction lies in its meridian plane. Furthermore, let the equator of the sensing element of the compass be located, at the initial instant, in a plane perpendicular to the force of attraction; let the vector of the kinetic moment of the gyrorotor lie in the meridian plane and let the hunting angle ε_0 of the gyroscopes be set by starting from the relation

$$2B \cos \varepsilon_0 = mlUR' \tag{1.1}$$

Moreover, if the moment N generated by the spring is subjected to the law

$$N = -\frac{4B^2}{mlR' / \sin \chi} \cos \varepsilon \sin \varepsilon \tag{1.2}$$

then the rotor will maintain the direction of the force of attraction and the meridian throughout the whole time of operation of the compass.

Here R' is the distance from the center of buoyancy of the gyrorotor 0 to the axis of rotation of the body; χ is the angle between the direction of the force of attraction and the axis of rotation of the body; the remaining notation is the same as in [2].

In the proof of this assertion, as everywhere henceforth, we shall start from the equations of motion of the gyrorotor relative to a coordinate system S moving progressively together with the body, but not taking part in its rotation, projected on the $Oxyz$-axes bound to the gyrorotor exactly as in [2]

$$\begin{aligned} -\omega_z 2B \cos \varepsilon &= M_x, \qquad \omega_x 2B \cos \varepsilon = M_z \\ \frac{d}{dt} (2B \cos \varepsilon) &= M_y, \qquad -\omega_y 2B \sin \varepsilon = N \end{aligned} \tag{1.3}$$

Here ω_x, ω_y and ω_z are the projections of the angular velocity of the gyrorotor (i.e. the trihedron $Oxyz$) relative to the system S on the $Oxyz$-axes, and M_x, M_y and M_z are the moments of the external forces applied to the gyrorotor and the inertial force of the translational motion.

It is assumed that the gyrorotor is oriented at the initial instant so that the z-axis is directly opposite to the direction of the force of attraction and the y-axis lies in the meridian plane. Let us assume that such an orientation is also maintained later. Then

$$\omega_x = 0, \qquad \omega_y = U \sin \chi, \qquad \omega_z = U \cos \chi$$
$$M_x = -mlU^2R' \cos \chi, \qquad M_y = 0, \quad M_z = 0$$

and equations (1.3), taking conditions (1.1) and (1.2) into account, are satisfied identically, which proves the assertion.

Conditions (1.1) and (1.2) differ from the conditions of Ishlinskii [2] only by the fact that the quantity $R'/\sin\chi$, the distance from the point of suspension of the gyrorotor to the axis of rotation of the earth along the line of action of the force of attraction, replaces the radius of the earth R in (1.2). This difference is very insignificant and, as will be shown below, the Geckeler-Ishlinskii gyrohorizon on a fixed base indicates the direction of the central force with a very high degree of accuracy (compare with the opposite statement in [1]).

Furthermore, the close connection between the direction of the force of attraction and the orientation of the gyrorotor once again confirms the naturalness and expediency of using formula (1.1) of [1] in studying the operation of a gyrohorizon, i.e. the method of taking account of the forces acting on a material system moving near the earth which Ishlinskii proposed.

However, the necessity arises here for a still clearer definition of all the concepts associated with the direction of the gravity force and the force of attraction at a given point than was given in [1].

2. In [1], the concept of a pseudovertical as "a line connecting a point on the surface with the center of the earth" is introduced together with the true vertical which coincides with the plumb line, i.e. the normal to the surface of the earth. The central force is hence considered to act along the pseudovertical, although it is agreed that such an assumption is only a first approximation. Correspondingly, the angle between the vertical and the pseudovertical equals $1/2(R_1U^2/g)\sin 2\varphi$ (formula (1.6) of [1]) and, in this connection, it is stated in particular that the axis of a gyrocompass with a Schuler period on a fixed base "indicates the pseudohorizontal plane exactly as does the gyrohorizon of Ishlinskii".

Let us note, however, that the angle between the direction of the force of attraction and the direction to the center of the earth (the pseudovertical in the Merkin terminology) is a quantity of the same order as is the angle between the vertical and the pseudovertical. If it is assumed that the surface of the earth has the shape of a Clairaut ellipsoid, then the angle between the vertical and the direction to the center of the earth is $\psi = \varphi - \varphi'$, where φ is the geographic and φ' the geocentric latitude of the locality and

$$\tan\psi = \frac{e^2\cos\varphi'\sin\varphi'}{1-e^2\cos^2\varphi'} \qquad \left(e^2 = \frac{a^2-b^2}{a^2}\right)$$

Here $e^2 = 0.0067$ is the square of the first eccentricity of the terrestrial meridian. To the accuracy of higher order infinitesimals than e^2, it can be considered that

$$\psi = {}^1/_2\, e^2 \sin 2\varphi' \tag{2.1}$$

Furthermore, if it is assumed that the earth is an ellipsoid of revolution of constant density, then by starting from the theory of the Newtonian potential [6-8], the projection of the terrestrial gravity vector in the direction to the center of the earth will be

$$g = \frac{4}{3}\pi f D \frac{a^2}{b} \frac{b^2}{R^2} \tag{2.2}$$

and on a perpendicular plane

$$g_1 = \frac{4}{5}\pi f D \frac{a^2 (a^2 - b^2)}{b^3} \frac{b^4}{R^4} \sin\varphi' \cos\varphi' \tag{2.3}$$

Here $f = 6.66 \times 10^{-8}$ $cm^3 g^{-1} sec^{-2}$ is the gravitational constant, $D = 15.52\ g\,cm^{-3}$ is the mean density of the terrestrial spheroid and R the distance from the center of the earth to a given point of its surface. Hence, the angle between the direction of the force of attraction and the direction to the center of the earth (the pseudovertical) is

$$\psi' \doteq \frac{3}{5} \frac{a^2 - b^2}{R^2} \sin\varphi' \cos\varphi'$$

i.e. to the same degree of accuracy (retaining small quantities on the order of e^2)

$$\psi' = {}^3/_5\, e^2 \sin\varphi' \cos\varphi' = {}^3/_5 \psi \tag{2.4}$$

Thus, taking account of the flattening of the earth requires a clear differentiation between the three directions at each point of the earth's surface: the direction to the center of the earth or the pseudovertical; the direction of the force of attraction or the lines of force of the earth's gravitational field, which we shall designate the gravitational vertical; and the force of gravity or the normal to the surface of the terrestrial spheroid, i.e. the true vertical.

The angle given by formula (1.6) in [1] is actually the angle between the gravitational and true verticals and is two-fifths of the angle between the pseudovertical and the vertical.

Within the scope of the assumptions on the spherical shape of the earth, the gravitational vertical coincides with the pseudovertical. Nevertheless, it should be stated that the gyrocompass with a Schuler period on a fixed base, just as the Geckeler gyrohorizon, shows the gravitational vertical, since the direction of the axes of these instruments is determined by precisely the direction of the force of attraction and not by the geometric direction to the center of the earth.

It was shown in Section 1 that for a deviation of the gravitational vertical from the pseudovertical, a gyrocompass very similar to the Geckeler gyrohorizon on a fixed base indicates the direction of the gravitational vertical.

In this connection, it is natural to consider a gyrocompass whose imperturbable sensing element retains the direction of the gravitational vertical and the plane of the meridian (to the accuracy of a course correction) for any motion of the base along the surface of the terrestrial spheroid.

3. Let us introduce the $O\xi\eta\zeta$ coordinate system in such a way that the ζ-axis is directed upward along the gravitational vertical, the ξ-axis eastward along the parallels and the η-axis forms a right trihedron with the ζ- and ξ-axes. This system transforms into the geographically oriented coordinate system [1] when a rotation through an angle 2/5 ψ is made around the ξ-axis. Correspondingly, the projections of the absolute velocity V of a point O moving along the surface of the terrestrial spheroid onto the $O\xi\eta\zeta$-axes will have the following form (to the accuracy of higher order infinitesimals than ψ):

$$V_\xi = v_E + RU\cos\varphi', \qquad V_\eta = v_N, \qquad V_\zeta = -{}^2/_5\psi v_N = -{}^2/_5\psi V_\eta \quad (3.1)$$

To this same accuracy (which we shall henceforth retain everywhere without stating this particularly) we will have

$$\dot{\varphi}' = \frac{V_\eta}{R}, \qquad \dot{\psi} = \psi\frac{V_\eta}{R} 2 \cot 2\varphi' \quad (3.2)$$

It is easy to see that the angular velocity of the trihedron $O\xi\eta\zeta$ has the projections

$$\omega_\xi = -\frac{V_\eta}{R}\left(1 + \frac{3}{5}\psi 2 \cot 2\varphi'\right)$$
$$\omega_\eta = \frac{V_\xi}{R}\left(1 - \frac{3}{5}\psi \tan\varphi'\right), \qquad \omega_\zeta = \frac{V_\xi}{R}\left(\tan\varphi' + \frac{3}{5}\psi\right) \quad (3.3)$$

Let us now introduce the $Ox^\circ y^\circ z^\circ$ coordinate system which is obtained from the $O\xi\eta\zeta$ system by a rotation around the ζ-axis through the angle $\vartheta = \tan^{-1}(V_\eta/V_\xi)$. Then the vector of the absolute velocity V of a point O will lie in the $Ox^\circ z^\circ$ plane and

$$V_{x^\circ} = V, \qquad V_{y^\circ} = 0, \qquad V_{z^\circ} = -{}^2/_5\psi V \sin\vartheta \quad (3.4)$$

The angular velocity of the trihedron $Ox^\circ y^\circ z^\circ$ has the projections

$$\omega_{x^\circ} = -\frac{3}{5}\psi\frac{V\sin 2\vartheta}{2R}\cot\varphi'$$

$$\omega_{y^\circ} = \frac{V}{R}\left[1 + \frac{3}{5}\psi\,(\sin^2\vartheta\,\cot\varphi' - \tan\varphi')\right] \tag{3.5}$$
$$\omega_{z^\circ} = \dot\vartheta + \frac{V\cos\vartheta}{R}\left(\tan\varphi' + \frac{3}{5}\psi\right) = \omega$$

The projections of the absolute acceleration w of the point O on the $Ox^\circ y^\circ z^\circ$-axes equal, respectively

$$w_{x^\circ} = \dot V_{x^\circ} + \omega_{y^\circ}V_{z^\circ} - \omega_{z^\circ}V_{y^\circ} = \dot V - \frac{2}{5}\psi\frac{V^2}{R}\sin\vartheta$$
$$w_{y_\circ} = V_{y^\circ} + \dot\omega_{z^\circ}V_{x^\circ} - \omega_{x^\circ}V_{z^\circ} = \omega V \tag{3.6}$$
$$w_{z^\circ} = \dot V_{z^\circ} + \omega_{x^\circ}V_{y^\circ} - \omega_{y^\circ}V_{x^\circ} =$$
$$= -\frac{V^2}{R} - \psi\left[\frac{V^2}{R}(\sin^2\vartheta\,\cot\varphi' - \tan\varphi') + \frac{2}{5}\dot V\sin\vartheta + \frac{2}{5}\omega V\cos\vartheta\right]$$

The force acting on the gyrorotor at its center of gravity is, in projections on the $Ox^\circ y^\circ z^\circ$-axes

$$Q_{x^\circ} = -mw_{x^\circ}, \qquad Q_{y^\circ} = -mw_{y^\circ}, \qquad Q_{z^\circ} = -m\,(g + w_{z^\circ}) \tag{3.7}$$

(Here g is not the acceleration due to gravity, but the acceleration of the gravitational force defined by formula (2.2).)

Associating the $Oxyz$ coordinate system with the $Ox^\circ y^\circ z^\circ$ trihedron by means of the angles α, β and γ, exactly as in [2], and substituting the appropriate expressions for the angular velocity of the trihedron $Oxyz$ and the moments of the force Q in equations (1.3), we obtain the equations of motion of the gyrorotor

$$2B\cos\varepsilon\,[\omega_{x^\circ}(\cos\alpha\sin\gamma + \sin\alpha\sin\beta\cos\gamma) + \omega_{y^\circ}(\sin\alpha\sin\gamma - \tag{3.8}$$
$$-\cos\alpha\sin\beta\cos\gamma) + (\omega + \dot\alpha)\cos\beta\cos\gamma + \dot\beta\sin\gamma] =$$
$$= ml\,[-w_{x^\circ}\sin\alpha\cos\beta + w_{y^\circ}\cos\alpha\cos\beta + (g + w_{z^\circ})\sin\beta] - M_x'$$
$$\frac{d}{dt}(2B\cos\varepsilon) = ml\,[w_{x^\circ}(\cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma) +$$
$$+ w_{y^\circ}(\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma) - (g + w_{z^\circ})\cos\beta\sin\gamma] + M_y'$$
$$(2B\cos\varepsilon\,[\omega_{x^\circ}(\cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma) + \omega_{y^\circ}(\sin\alpha\cos\gamma +$$
$$+ \cos\alpha\sin\beta\sin\gamma) - (\omega + \dot\alpha)\cos\beta\sin\gamma + \dot\beta\cos\gamma] = M_z'$$
$$2B\sin\varepsilon\,[-\omega_{x^\circ}\sin\alpha\cos\beta + \omega_{y^\circ}\cos\alpha\cos\beta + (\omega + \dot\alpha)\sin\beta + \dot\gamma] = -N$$

Setting $\alpha = \beta = \gamma = 0$ in these equations we obtain

$$2B\omega\cos\varepsilon = ml\omega V - M_x'$$

$$\frac{d}{dt}(2B\cos\varepsilon) = ml\dot{V} - ml\,\frac{2}{5}\,\psi\,\frac{V^2}{R}\sin\vartheta + M_y' \tag{3.9}$$
$$-\,2B\cos\varepsilon\,\frac{V}{R}\,\frac{3}{5}\psi\,\cot\varphi'\sin\vartheta\cos\vartheta = M_z'$$
$$2B\sin\varepsilon\,\frac{V}{R}\left[1+\frac{3}{5}\psi\,(\sin^2\vartheta\,\cot\varphi' - \tan\varphi')\right] = -\,N$$

Hence, if the moments

$$M_y' = ml\,\frac{2}{5}\,\psi\,\frac{V^2}{R}\sin\vartheta, \quad M_z' = -ml\,\frac{3}{5}\,\psi\,\frac{V^2}{R}\sin\vartheta\cos\vartheta\,\cot\varphi', \quad M_x' = 0 \tag{3.10}$$

act continuously on the gyrorotor and the spring moment changes according to the law

$$N = -\,\frac{4B^2\sin\varepsilon\cos\varepsilon}{mlR}\left[1-\frac{3}{5}\,\psi\,(\tan\varphi' - \sin^2\vartheta\,\cot\varphi')\right] \tag{3.11}$$

and the gyroscope hunting angle ε_0 is set at the initial instant according to the condition

$$\cos\varepsilon_0 = \frac{mlV_0}{2B} \tag{3.12}$$

then the gyroscope will indicate the direction of the gravitational vertical and the plane of the meridian to the accuracy of the course correction

$$\vartheta = \tan^{-1}\frac{v_N}{v_E + RU\cos\varphi'}$$

no matter what the maneuver of the base, if only $\alpha = \beta = \gamma = 0$ at the initial instant. The moments (3.10) are generated artificially and are imposed on the gyrorotor from without.

A rather different method of producing a gyrocompass which is not perturbable in the mentioned sense, wherein the additional moments are produced within the gyrorotor itself, is of interest. Up to now the center of gravity of the gyrorotor was assumed to be disposed fixedly at a distance $-l$ from the center of buoyancy in the z direction. Let us assume now that by using some kind of apparatus the center of gravity can be displaced by small distances within the gyrorotor in conformity with signals being sent from a certain computer. Then it seems that it is sufficient that the metacentric height l change according to the law

$$l = l_0\exp\left\{\frac{3}{5}\int_0^t \psi\,\frac{\sin\vartheta}{R}\left[\left(g-\frac{V^2}{R}\right)\frac{\cos\vartheta}{\omega}\,\cot\varphi' - \frac{2}{3}\,V\right]d\tau\right\} \tag{3.13}$$

and that the direction from the center of buoyancy toward the center of gravity be inclined with respect to the z-axis at the angle

$$\theta = \frac{3}{5}\psi \frac{V \sin 2\vartheta}{2R\omega} \cot \varphi' \tag{3.14}$$

in the xz-plane. Upon compliance with conditions (3.13) and (3.14) together with (3.11) and (3.12) and for zero initial conditions, the sensing element of the gyrohorizon will be oriented continuously along the gravitational vertical and the plane of the meridian (taking account of the course deviation). The correctness of this statement follows directly from (3.8), upon substitution of the appropriate additional moments which arise during such a deviation of the center of gravity from the z-axis

$$M_x' = 0$$

$$M_y' = -\theta ml\, [w_{x^\circ}(\cos\alpha \sin\gamma + \sin\alpha \sin\beta\cos\gamma) + w_{y^\circ}(\sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma) + (g + w_{z^\circ})\cos\beta\cos\gamma]$$

$$M_z' = \theta ml\, [w_{x^\circ}\sin\alpha\cos\beta + w_{y^\circ}\cos\alpha\sin\beta + (g + w_{z^\circ})\sin\beta]$$

Hence, in principle, it is possible to construct a gyrohorizon which will take the nonsphericity of the earth into account with the accuracy of the eccentricity squared e^2, and which will constantly indicate either the gravitational vertical or the true vertical [1] and the meridian plane with the accuracy of a course correction. However, in connection with the remarks made in Section 1, it is interesting to estimate the deviation from the gravitational vertical and from the gyro-north of the Geckeler-Ishlinskii gyrohorizon [2], associated with the flattening of the figure of the earth.

4. Let us transform to the equations of small vibrations by assuming α, β, γ and $\delta = \varepsilon - \sigma(t)$, where $\sigma(t)$ satisfies the condition $2B\cos\sigma = ml$ and their derivatives are first order infinitesimals, exactly as is ψ, and higher order infinitesimals are discarded. We then obtain

$$\frac{d}{dt}(\varkappa + \mu) + i(\nu - \omega)(\varkappa + \mu) = +\frac{3}{5}\psi\frac{V}{R}\Big[\tan\varphi' - \sin^2\vartheta \cot\varphi' + i\sin\vartheta\Big(\cos\vartheta \cot\varphi' + \frac{2}{3}\frac{V}{\nu R}\Big)\Big] \qquad \Big(\varkappa = \frac{V}{\sqrt{gR}}\alpha + i\beta\Big) \tag{4.1}$$

$$\frac{d}{dt}(\varkappa - \mu) + i(\nu + \omega)(\varkappa - \mu) = \frac{3}{5}\psi\frac{V}{R}\Big[-\tan\psi' + \sin^2\vartheta \cot\psi' + i\sin\vartheta\Big(\cos\vartheta \cot\psi' - \frac{2}{3}\frac{V}{\nu R}\Big)\Big] \qquad \Big(\mu = \gamma - i\frac{2B\sin\sigma}{ml\sqrt{gR}}\delta\Big)$$

These equations differ from equations (5.3) of [2] by the presence of non-zero right-hand sides which depend on the velocity of the craft, the course and the latitude.

In the case of a fixed base, we have small vibrations around the constant values

$$\alpha_0 = 0, \quad \beta_0 = -\frac{3}{5}\psi\frac{\omega}{\nu^2-\omega^2}\frac{V}{R}\tan\varphi', \quad \gamma_0 = 0, \quad \delta_0 = \frac{3}{5}\psi\frac{\nu^2}{\nu^2-\omega^2}\frac{\tan\varphi'}{\tan\sigma} \tag{4.2}$$

Since $V = UR\cos\varphi'$, $\omega = U\sin\varphi' + O(\psi)$ and taking into account that $2/5\,\psi = 1/2(RU^2/g)\sin 2\varphi'$ (the angle between the vertical and the gravitational vertical) and therefore

$$\nu^2 = \frac{5}{2}\frac{U^2}{e^2} \tag{4.3}$$

we obtain that the axis of the gyrorotor on a fixed base deviates from the gravitational vertical by the angle

$$\beta_0 = -\,{}^6/_{25}\, e^4 \sin^3\varphi' \cos\varphi' \tag{4.4}$$

The maximum deviation occurs at a $\varphi' = 60^\circ$ latitude and is $\beta_{0\,max} = -\,0.7''$ for a 4′ angle between the true and gravitational verticals. The relative error in measuring the angle between the true and gravitational verticals by using the gyrohorizon is determined by the expression

$$\frac{|\beta_0|}{{}^2/_5\psi} = \frac{3}{5}e^2\sin^2\varphi' \tag{4.5}$$

and reaches a maximum value of 0.004. Hence, the Geckeler gyrohorizon indicates the direction of the gravitational vertical, i.e. the direction of the force of attraction at a given point of the earth, with a sufficiently high degree of accuracy on a fixed base.

Equations (4.1) permit an estimate of the drift of a gyrohorizon on a moving base also. Thus, for a constant velocity at a constant course, we have (neglecting the very slow change in latitude)

$$\begin{aligned}
\alpha_0 &= \frac{3}{5}\psi\frac{\nu}{\nu^2-\omega^2}\sin\vartheta\left(\nu\cos\vartheta\cot\varphi' + \frac{2}{3}\frac{\omega}{\nu}\frac{V}{R}\right)\\
\beta_0 &= -\frac{3}{5}\psi\frac{\omega}{\nu^2-\omega^2}\frac{V}{R}(\tan\varphi' - \sin^2\vartheta\cot\varphi')\\
\gamma_0 &= \frac{3}{5}\psi\frac{1}{\nu^2-\omega^2}\frac{V}{R}\sin\vartheta\left(\omega\cos\vartheta\cot\varphi' + \frac{2}{3}\frac{V}{R}\right)\\
\delta_0 &= \frac{3}{5}\psi\frac{\nu^2}{\nu^2-\omega^2}\frac{1}{\tan\sigma}(\tan\varphi' - \sin^2\vartheta\cot\varphi')
\end{aligned} \tag{4.6}$$

For example, let a craft move on a northward course at a velocity of 20 knots at a 60° latitude. Then $\alpha_0 = 8''$, $\beta_0 = 0.8''$ and $\gamma_0 = 0.02''$. The quantity δ_0 depends on the angle σ. If the parameters of the gyrorotor are selected and that $\sigma \approx \psi$, then $\delta_0 \approx 3/5\,\psi = 5.9'$.

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